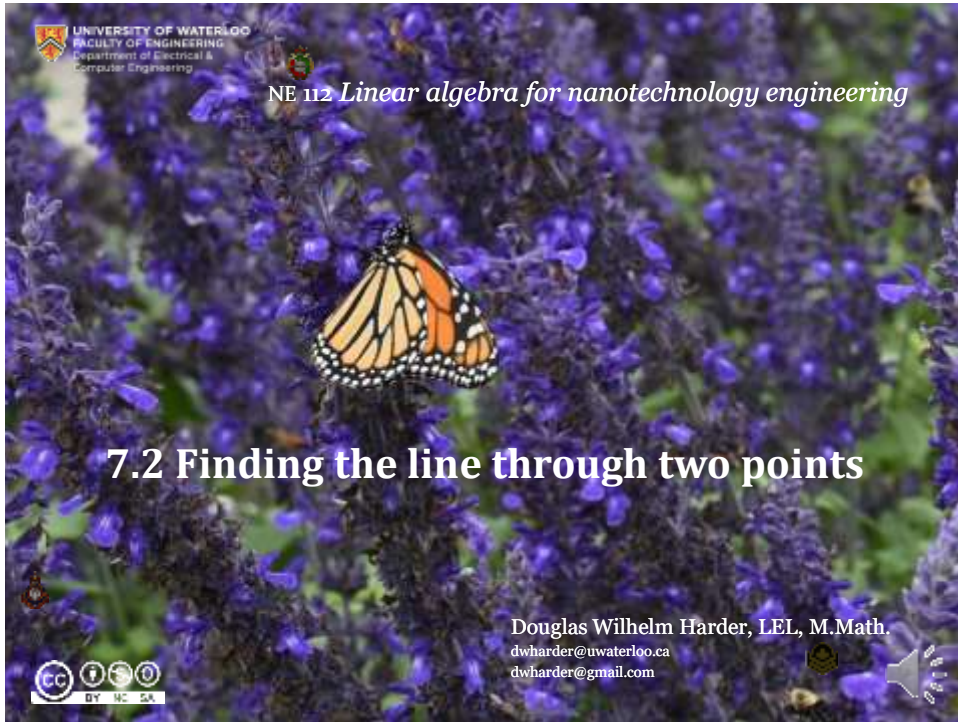




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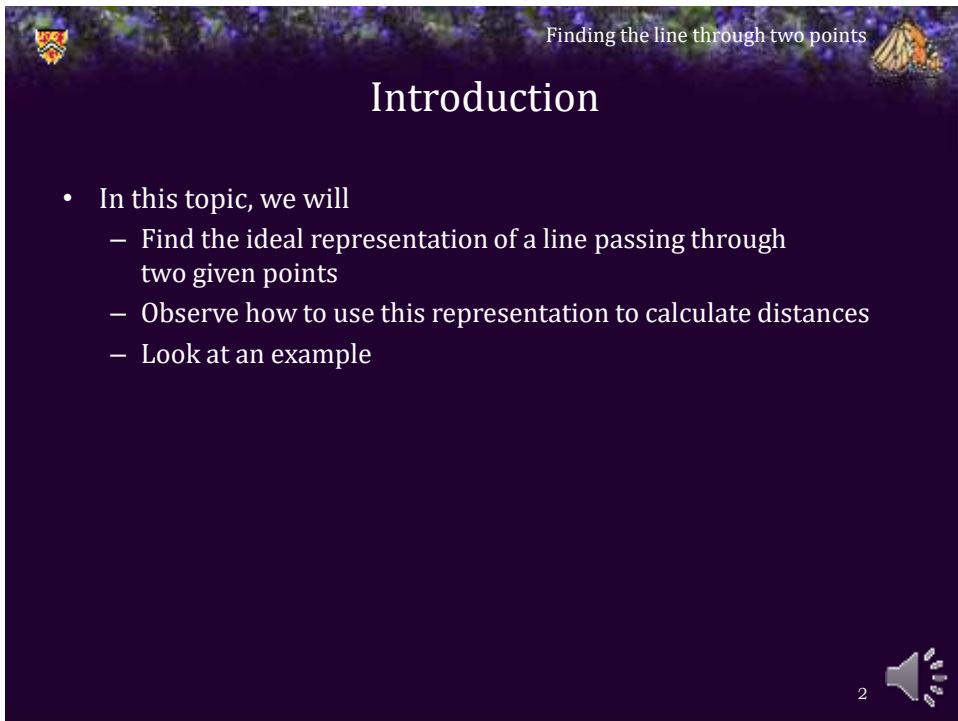
NE 112 *Linear algebra for nanotechnology engineering*

## 7.2 Finding the line through two points

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Finding the line through two points

## Introduction

- In this topic, we will
  - Find the ideal representation of a line passing through two given points
  - Observe how to use this representation to calculate distances
  - Look at an example

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## $\mathbf{R}^3$

- In  $\mathbf{R}^3$ , given two points  $\mathbf{u}_1$  and  $\mathbf{u}_2$ , the line that passes through these two points may be found as follows:
  - Calculate
 
$$\hat{\mathbf{v}} \leftarrow \frac{\mathbf{u}_2 - \mathbf{u}_1}{\|\mathbf{u}_2 - \mathbf{u}_1\|_2}$$
  - Next, choose that point  $\mathbf{u}_1$  or  $\mathbf{u}_2$  that has the smallest 2-norm
    - That is, pick the one already closer to the origin
 
$$\mathbf{u}_\perp \leftarrow \mathbf{u}_k - \langle \hat{\mathbf{v}}, \mathbf{u}_k \rangle \hat{\mathbf{v}}$$

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## $\mathbf{R}^3$

- For example, find the line through the points
 
$$\mathbf{u}_1 = \begin{pmatrix} 18 \\ 14 \\ -54 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 36 \\ -26 \\ -30 \end{pmatrix}$$
  - First, find  $\hat{\mathbf{v}}$ :
 
$$\hat{\mathbf{v}} = \frac{\mathbf{u}_2 - \mathbf{u}_1}{\|\mathbf{u}_2 - \mathbf{u}_1\|_2} = \begin{pmatrix} 0.36 \\ -0.8 \\ 0.48 \end{pmatrix}$$
  - Next, because  $\|\mathbf{u}_2\|_2 < \|\mathbf{u}_1\|_2$ ,
 
$$\mathbf{u}_\perp \leftarrow \begin{pmatrix} 36 \\ -26 \\ -30 \end{pmatrix} - \left\langle \hat{\mathbf{v}}, \begin{pmatrix} 36 \\ -26 \\ -30 \end{pmatrix} \right\rangle \hat{\mathbf{v}} = \begin{pmatrix} 29.0304 \\ -10.512 \\ -39.2928 \end{pmatrix}$$

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$\mathbf{R}^3$

- Consequently, the ideal representation of the line passing through
 
$$\mathbf{u}_1 = \begin{pmatrix} 18 \\ 14 \\ -54 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 36 \\ -26 \\ -30 \end{pmatrix}$$
 is
 
$$\begin{pmatrix} 29.0304 \\ -10.512 \\ -39.2928 \end{pmatrix} + \alpha \begin{pmatrix} 0.36 \\ -0.8 \\ 0.48 \end{pmatrix}$$

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$\mathbf{R}^3$

- You can also determine how far away each of the two points is from this closest point:
 
$$\langle \hat{\mathbf{v}}, \mathbf{u}_1 \rangle = -30.64$$

$$\langle \hat{\mathbf{v}}, \mathbf{u}_2 \rangle = 19.36$$
- Thus, the distance between the two points is 50 m, and we observe that  $\|\mathbf{u}_2 - \mathbf{u}_1\|_2 = 50$

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## Summary

- Following this topic, you now
  - Know how to find the ideal vector representation of a line passing through two points
  - Understand that it is best, when given a choice, to use a vector with the smallest 2-norm when calculating perpendicular components



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## References

[1] [https://en.wikipedia.org/wiki/Line\\_\(geometry\)](https://en.wikipedia.org/wiki/Line_(geometry))



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# Acknowledgments

None so far.

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# Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see <https://www.rbg.ca/> for more information.



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